

In[4]:= **ClearAll**[f, I0, x0, x, Ib, G];
f[x_] = (2 BesselJ[1, G*x] / (G*x))^2

Out[5]=
$$\frac{4 \text{BesselJ}[1, Gx]^2}{G^2 x^2}$$

In[6]:= **TeXForm**[f[x]]

Out[6]/TeXForm=
$$\frac{4 J_1(Gx)^2}{G^2 x^2}$$

In[7]:= **Isp = 1 / (2 b) Integrate**[f[x-y], {y, -b, b},
Assumptions -> {b > 0, G > 0, Element[{b, G, x}, Reals]}]

Out[7]= **ConditionalExpression**
$$\left[\frac{1}{3 b G^2 (b-x) (b+x)} \right.$$

$$2 \left(2 G^2 (b-x)^2 (b+x) \text{BesselJ}[0, G (b-x)]^2 + 2 G^2 (b-x) (b+x)^2 \text{BesselJ}[0, G (b+x)]^2 + \right.$$

$$2 G (-b^2 + x^2) \text{BesselJ}[0, G (b-x)] \text{BesselJ}[1, G (b-x)] -$$

$$b \text{BesselJ}[1, G (b-x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G (b-x)]^2 -$$

$$x \text{BesselJ}[1, G (b-x)]^2 - 2 b^2 G^2 x \text{BesselJ}[1, G (b-x)]^2 -$$

$$2 b G^2 x^2 \text{BesselJ}[1, G (b-x)]^2 + 2 G^2 x^3 \text{BesselJ}[1, G (b-x)]^2 +$$

$$2 G (-b^2 + x^2) \text{BesselJ}[0, G (b+x)] \text{BesselJ}[1, G (b+x)] -$$

$$b \text{BesselJ}[1, G (b+x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G (b+x)]^2 +$$

$$x \text{BesselJ}[1, G (b+x)]^2 + 2 b^2 G^2 x \text{BesselJ}[1, G (b+x)]^2 -$$

$$2 b G^2 x^2 \text{BesselJ}[1, G (b+x)]^2 - 2 G^2 x^3 \text{BesselJ}[1, G (b+x)]^2 \left. \right),$$

$$(b < x \ \&\& \ x > 0) \ || \ (x < 0 \ \&\& \ b + x < 0) \left. \right]$$

In[8]:= **Isp = FullSimplify**[Isp]

Out[8]= **ConditionalExpression**
$$\left[\frac{1}{3 b} \right.$$

$$2 \left(2 (b-x) \text{BesselJ}[0, G (b-x)]^2 + 2 (b+x) \text{BesselJ}[0, G (b+x)]^2 - \right.$$

$$\frac{2 \text{BesselJ}[0, G (b-x)] \text{BesselJ}[1, G (b-x)]}{G} -$$

$$\frac{2 \text{BesselJ}[0, G (b+x)] \text{BesselJ}[1, G (b+x)]}{G} +$$

$$\left. \frac{\frac{(-1+2 G^2 (b-x)^2) \text{BesselJ}[1, G (b-x)]^2}{b-x} + \frac{(-1+2 G^2 (b+x)^2) \text{BesselJ}[1, G (b+x)]^2}{b+x}}{G^2} \right), b < x \ || \ b + x < 0 \left. \right]$$

erfordert $b < x$ oder $b < -x$ d.h gilt nicht für $-b \leq x \leq b$

Test für Bereich $x < -b$

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In[9]:= Isp1 = 1 / (2 b) Integrate[f[x-y], {y, -b, b},
      Assumptions -> {b > 0, Element[{b, G, x}, Reals], x < -b}]
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$$\text{Out[9]= } \frac{1}{3 b G^2 (b-x) (b+x)} 2 \left(2 G^2 (b-x)^2 (b+x) \text{BesselJ}[0, G (b-x)]^2 + 2 G^2 (b-x) (b+x)^2 \text{BesselJ}[0, G (b+x)]^2 + 2 G (-b^2 + x^2) \text{BesselJ}[0, G (b-x)] \text{BesselJ}[1, G (b-x)] - b \text{BesselJ}[1, G (b-x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G (b-x)]^2 - x \text{BesselJ}[1, G (b-x)]^2 - 2 b^2 G^2 x \text{BesselJ}[1, G (b-x)]^2 - 2 b G^2 x^2 \text{BesselJ}[1, G (b-x)]^2 + 2 G^2 x^3 \text{BesselJ}[1, G (b-x)]^2 + 2 G (-b^2 + x^2) \text{BesselJ}[0, G (b+x)] \text{BesselJ}[1, G (b+x)] - b \text{BesselJ}[1, G (b+x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G (b+x)]^2 + x \text{BesselJ}[1, G (b+x)]^2 + 2 b^2 G^2 x \text{BesselJ}[1, G (b+x)]^2 - 2 b G^2 x^2 \text{BesselJ}[1, G (b+x)]^2 - 2 G^2 x^3 \text{BesselJ}[1, G (b+x)]^2 \right)$$

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In[10]:= Isp1 = FullSimplify[Isp1]
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$$\text{Out[10]= } \frac{1}{3 b} 2 \left(2 (b-x) \text{BesselJ}[0, G (b-x)]^2 + 2 (b+x) \text{BesselJ}[0, G (b+x)]^2 - \frac{2 \text{BesselJ}[0, G (b-x)] \text{BesselJ}[1, G (b-x)]}{G} - \frac{2 \text{BesselJ}[0, G (b+x)] \text{BesselJ}[1, G (b+x)]}{G} + \frac{\frac{(-1+2 G^2 (b-x)^2) \text{BesselJ}[1, G (b-x)]^2}{b-x} + \frac{(-1+2 G^2 (b+x)^2) \text{BesselJ}[1, G (b+x)]^2}{b+x}}{G^2} \right)$$

Test für Bereich $-b \leq x < 0$

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In[11]:= Isp2 = 1 / (2 b) Integrate[f[x-y], {y, -b, b},
      Assumptions -> {b > 0, Element[{b, G, x}, Reals], -b <= x, x < 0}]
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$$\text{Out[11]= } \frac{1}{3 b G^2 (b-x) (b+x)} 2 \left(2 G^2 (b-x)^2 (b+x) \text{BesselJ}[0, G (b-x)]^2 + 2 G^2 (b-x) (b+x)^2 \text{BesselJ}[0, G (b+x)]^2 + 2 G (-b^2 + x^2) \text{BesselJ}[0, G (b-x)] \text{BesselJ}[1, G (b-x)] - b \text{BesselJ}[1, G (b-x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G (b-x)]^2 - x \text{BesselJ}[1, G (b-x)]^2 - 2 b^2 G^2 x \text{BesselJ}[1, G (b-x)]^2 - 2 b G^2 x^2 \text{BesselJ}[1, G (b-x)]^2 + 2 G^2 x^3 \text{BesselJ}[1, G (b-x)]^2 + 2 G (-b^2 + x^2) \text{BesselJ}[0, G (b+x)] \text{BesselJ}[1, G (b+x)] - b \text{BesselJ}[1, G (b+x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G (b+x)]^2 + x \text{BesselJ}[1, G (b+x)]^2 + 2 b^2 G^2 x \text{BesselJ}[1, G (b+x)]^2 - 2 b G^2 x^2 \text{BesselJ}[1, G (b+x)]^2 - 2 G^2 x^3 \text{BesselJ}[1, G (b+x)]^2 \right)$$

In[12]:= **Isp2 = FullSimplify[Isp2]**

$$\text{Out[12]} = \frac{1}{3b} 2 \left(2 (b-x) \text{BesselJ}[0, G (b-x)]^2 + \right.$$

$$2 (b+x) \text{BesselJ}[0, G (b+x)]^2 - \frac{2 \text{BesselJ}[0, G (b-x)] \text{BesselJ}[1, G (b-x)]}{G} -$$

$$\frac{2 \text{BesselJ}[0, G (b+x)] \text{BesselJ}[1, G (b+x)]}{G} +$$

$$\left. \frac{\frac{(-1+2 G^2 (b-x)^2) \text{BesselJ}[1, G (b-x)]^2}{b-x} + \frac{(-1+2 G^2 (b+x)^2) \text{BesselJ}[1, G (b+x)]^2}{b+x}}{G^2} \right)$$

Test für Bereich $0 < x \leq b$

In[13]:= **Isp3 = 1 / (2 b) Integrate[f[x-y], {y, -b, b},**
Assumptions -> {b > 0, Element[{b, G, x}, Reals], 0 < x, x <= b}]

$$\text{Out[13]} = \frac{1}{3 b G^2 (b-x) (b+x)}$$

$$2 \left(2 G^2 (b-x)^2 (b+x) \text{BesselJ}[0, G (b-x)]^2 + 2 G^2 (b-x) (b+x)^2 \text{BesselJ}[0, G (b+x)]^2 + \right.$$

$$2 G (-b^2 + x^2) \text{BesselJ}[0, G (b-x)] \text{BesselJ}[1, G (b-x)] -$$

$$b \text{BesselJ}[1, G (b-x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G (b-x)]^2 -$$

$$x \text{BesselJ}[1, G (b-x)]^2 - 2 b^2 G^2 x \text{BesselJ}[1, G (b-x)]^2 -$$

$$2 b G^2 x^2 \text{BesselJ}[1, G (b-x)]^2 + 2 G^2 x^3 \text{BesselJ}[1, G (b-x)]^2 +$$

$$2 G (-b^2 + x^2) \text{BesselJ}[0, G (b+x)] \text{BesselJ}[1, G (b+x)] -$$

$$b \text{BesselJ}[1, G (b+x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G (b+x)]^2 +$$

$$x \text{BesselJ}[1, G (b+x)]^2 + 2 b^2 G^2 x \text{BesselJ}[1, G (b+x)]^2 -$$

$$\left. 2 b G^2 x^2 \text{BesselJ}[1, G (b+x)]^2 - 2 G^2 x^3 \text{BesselJ}[1, G (b+x)]^2 \right)$$

In[14]:= **Isp3 = FullSimplify[Isp3]**

$$\text{Out[14]} = \frac{1}{3b} 2 \left(2 (b-x) \text{BesselJ}[0, G (b-x)]^2 + \right.$$

$$2 (b+x) \text{BesselJ}[0, G (b+x)]^2 - \frac{2 \text{BesselJ}[0, G (b-x)] \text{BesselJ}[1, G (b-x)]}{G} -$$

$$\frac{2 \text{BesselJ}[0, G (b+x)] \text{BesselJ}[1, G (b+x)]}{G} +$$

$$\left. \frac{\frac{(-1+2 G^2 (b-x)^2) \text{BesselJ}[1, G (b-x)]^2}{b-x} + \frac{(-1+2 G^2 (b+x)^2) \text{BesselJ}[1, G (b+x)]^2}{b+x}}{G^2} \right)$$

Test für Bereich $x > b$

In[15]:= **Isp4 = 1 / (2 b) Integrate[f[x - y], {y, -b, b},
Assumptions → {b > 0, Element[{b, G, x}, Reals], x > b}]**

$$\text{Out[15]= } \frac{1}{3 b G^2 (b-x)(b+x)} \left(2 \left(2 G^2 (b-x)^2 (b+x) \text{BesselJ}[0, G(b-x)]^2 + 2 G^2 (b-x)(b+x)^2 \text{BesselJ}[0, G(b+x)]^2 + \right. \right. \\ \left. \left. 2 G (-b^2 + x^2) \text{BesselJ}[0, G(b-x)] \text{BesselJ}[1, G(b-x)] - \right. \right. \\ \left. \left. b \text{BesselJ}[1, G(b-x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G(b-x)]^2 - \right. \right. \\ \left. \left. x \text{BesselJ}[1, G(b-x)]^2 - 2 b^2 G^2 x \text{BesselJ}[1, G(b-x)]^2 - \right. \right. \\ \left. \left. 2 b G^2 x^2 \text{BesselJ}[1, G(b-x)]^2 + 2 G^2 x^3 \text{BesselJ}[1, G(b-x)]^2 + \right. \right. \\ \left. \left. 2 G (-b^2 + x^2) \text{BesselJ}[0, G(b+x)] \text{BesselJ}[1, G(b+x)] - \right. \right. \\ \left. \left. b \text{BesselJ}[1, G(b+x)]^2 + 2 b^3 G^2 \text{BesselJ}[1, G(b+x)]^2 + \right. \right. \\ \left. \left. x \text{BesselJ}[1, G(b+x)]^2 + 2 b^2 G^2 x \text{BesselJ}[1, G(b+x)]^2 - \right. \right. \\ \left. \left. 2 b G^2 x^2 \text{BesselJ}[1, G(b+x)]^2 - 2 G^2 x^3 \text{BesselJ}[1, G(b+x)]^2 \right)$$

In[16]:= **Isp4 = FullSimplify[Isp4]**

$$\text{Out[16]= } \frac{1}{3 b} \left(2 (b-x) \text{BesselJ}[0, G(b-x)]^2 + \right. \\ \left. 2 (b+x) \text{BesselJ}[0, G(b+x)]^2 - \frac{2 \text{BesselJ}[0, G(b-x)] \text{BesselJ}[1, G(b-x)]}{G} - \right. \\ \left. \frac{2 \text{BesselJ}[0, G(b+x)] \text{BesselJ}[1, G(b+x)]}{G} + \right. \\ \left. \frac{\frac{(-1+2 G^2 (b-x)^2) \text{BesselJ}[1, G(b-x)]^2}{b-x} + \frac{(-1+2 G^2 (b+x)^2) \text{BesselJ}[1, G(b+x)]^2}{b+x}}{G^2} \right)$$

Test für Bereich $x = 0$

In[17]:= **Isp5 = 1 / (2 b) Integrate[f[x - y], {y, -b, b},
Assumptions → {b > 0, G > 0, Element[{b, G, x}, Reals], x == 0}]**

$$\text{Out[17]= } \frac{1}{3 b G^2} \left(2 b G^2 \text{BesselJ}[0, b G]^2 - \right. \\ \left. 2 G \text{BesselJ}[0, b G] \text{BesselJ}[1, b G] + \frac{(-1 + 2 b^2 G^2) \text{BesselJ}[1, b G]^2}{b} \right)$$

In[18]:= **Isp5 = FullSimplify[Isp5]**

$$\text{Out[18]= } \frac{8}{3} \text{BesselJ}[0, b G]^2 - \frac{8 \text{BesselJ}[0, b G] \text{BesselJ}[1, b G]}{3 b G} + \left(\frac{8}{3} - \frac{4}{3 b^2 G^2} \right) \text{BesselJ}[1, b G]^2$$

Tests auf Gleichheit

In[19]:= **Isp2 == Isp1**

Out[19]= True

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In[20]:= Isp2 - Isp1
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Out[20]= 0
```

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In[21]:= {Isp3 == Isp1, Isp3 == Isp2}
```

```
Out[21]= {True, True}
```

```
In[22]:= {Isp3 - Isp1, Isp3 - Isp2}
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```
Out[22]= {0, 0}
```

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In[23]:= {Isp4 == Isp3, Isp4 == Isp2, Isp4 == Isp1}
```

```
Out[23]= {True, True, True}
```

```
In[24]:= {Isp4 - Isp3, Isp4 - Isp2, Isp4 - Isp1}
```

```
Out[24]= {0, 0, 0}
```

```
In[25]:= x = 0; FullSimplify[Isp5 - Isp1]
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```
Out[25]= 0
```